

Dear students (IXB) Science(Physics))  
Teacher : BIPLAB DAS  
Good morning .

Study materials for today (19th May,  
2020) .

Go through the followings :

1. Topic 9.4.1 Mathematical Formulation of 2nd Law of Motion and go through the examples 9.1 to examples 9.5 ( pages 119 to 121) .
2. The video clips for the examples of Inertia and the Second law of motion .

After that write the following questions :

1. Go through the video clip on Mathematical Formulation of Second Law of Motion .
2. Write the Mathematical formulation of Second Law of motion .
3. Practice the examples from the book ( Examples 9.1 to 9.5 ) .

Home work will be uploaded by 6 pm  
( Wednesday, 20th May) .

That's all for today . Thanks . 12:18 am ✓

object, its velocity changes, that is, the object gets an acceleration. We would now like to study how the acceleration of an object depends on the force applied to it and how we measure a force. Let us recount some observations from our everyday life. During the game of table tennis if the ball hits a player it does not hurt him. On the other hand, when a fast moving cricket ball hits a spectator, it may hurt him. A truck at rest does not require any attention when parked along a roadside. But a moving truck, even at speeds as low as  $5 \text{ m s}^{-1}$ , may kill a person standing in its path. A small mass, such as a bullet may kill a person when fired from a gun. These observations suggest that the impact produced by the objects depends on their mass and velocity. Similarly, if an object is to be accelerated, we know that a greater force is required to give a greater velocity. In other words, there appears to exist some quantity of importance that combines the object's mass and its velocity. One such property called momentum was introduced by Newton. The momentum,  $p$  of an object is defined as the product of its mass,  $m$  and velocity,  $v$ . That is,

$$p = mv \quad (9.1)$$

Momentum has both direction and magnitude. Its direction is the same as that of velocity,  $v$ . The SI unit of momentum is kilogram-metre per second ( $\text{kg m s}^{-1}$ ). Since the application of an unbalanced force brings a change in the velocity of the object, it is therefore clear that a force also produces a change of momentum.

Let us consider a situation in which a car with a dead battery is to be pushed along a straight road to give it a speed of  $1 \text{ m s}^{-1}$ , which is sufficient to start its engine. If one or two persons give a sudden push (unbalanced force) to it, it hardly starts. But a continuous push over some time results in a gradual acceleration of the car to this speed. It means that the change of momentum of the car is not only determined by the magnitude of the force but also by the time during which the force is exerted. It may then also be concluded that the force necessary to

change the momentum of an object depends on the time rate at which the momentum is changed.

The second law of motion states that the rate of change of momentum of an object is proportional to the applied unbalanced force in the direction of force.

### 9.4.1 MATHEMATICAL FORMULATION OF SECOND LAW OF MOTION

Suppose an object of mass,  $m$  is moving along a straight line with an initial velocity,  $u$ . It is uniformly accelerated to velocity,  $v$  in time,  $t$  by the application of a constant force,  $F$  throughout the time,  $t$ . The initial and final momentum of the object will be,  $p_1 = mu$  and  $p_2 = mv$  respectively.

$$\begin{aligned} \text{The change in momentum} &\propto p_2 - p_1 \\ &\propto mv - mu \\ &\propto m \times (v - u). \end{aligned}$$

$$\text{The rate of change of momentum} \propto \frac{m \times (v - u)}{t}$$

Or, the applied force,

$$F \propto \frac{m \times (v - u)}{t}$$

$$F = \frac{km \times (v - u)}{t} \quad (9.2)$$

$$= kma \quad (9.3)$$

Here  $a [= (v - u)/t]$  is the acceleration, which is the rate of change of velocity. The quantity,  $k$  is a constant of proportionality. The SI units of mass and acceleration are  $\text{kg}$  and  $\text{m s}^{-2}$  respectively. The unit of force is so chosen that the value of the constant,  $k$  becomes one. For this, one unit of force is defined as the amount that produces an acceleration of  $1 \text{ m s}^{-2}$  in an object of  $1 \text{ kg}$  mass. That is,

$$1 \text{ unit of force} = k \times (1 \text{ kg}) \times (1 \text{ m s}^{-2}).$$

Thus, the value of  $k$  becomes 1. From Eq. (9.3)

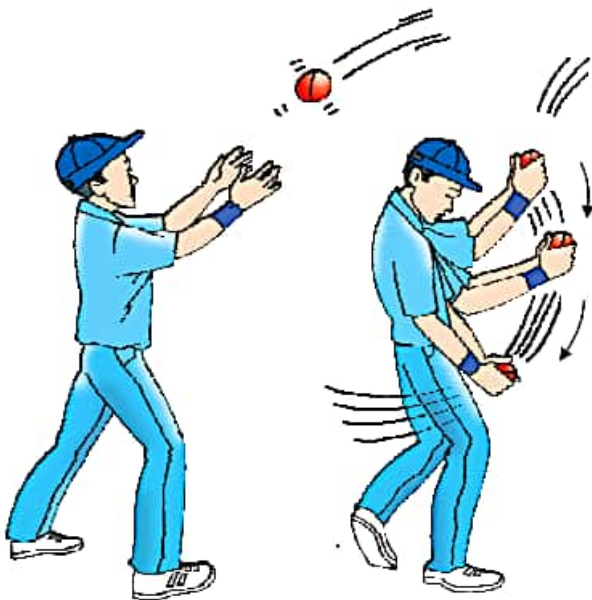
$$F = ma \quad (9.4)$$

The unit of force is  $\text{kg m s}^{-2}$  or newton, which has the symbol  $\text{N}$ . The second law of



motion gives us a method to measure the force acting on an object as a product of its mass and acceleration.

The second law of motion is often seen in action in our everyday life. Have you noticed that while catching a fast moving cricket ball, a fielder in the ground gradually pulls his hands backwards with the moving ball? In doing so, the fielder increases the time during which the high velocity of the moving ball decreases to zero. Thus, the acceleration of the ball is decreased and therefore the impact of catching the fast moving ball (Fig. 9.8) is also reduced. If the ball is stopped suddenly then its high velocity decreases to zero in a very short interval of time. Thus, the rate of change of momentum of the ball will be large. Therefore, a large force would have to be applied for holding the catch that may hurt the palm of the fielder. In a high jump athletic event, the athletes are made to fall either on a cushioned bed or on a sand bed. This is to increase the time of the athlete's fall to stop after making the jump. This decreases the rate of change of momentum and hence the force. Try to ponder how a karate player breaks a slab of ice with a single blow.



**Fig. 9.8:** A fielder pulls his hands gradually with the moving ball while holding a catch.

The first law of motion can be mathematically stated from the mathematical expression for the second law of motion. Eq. (9.4) is

$$F = ma$$

$$\text{or } F = \frac{m(v-u)}{t} \quad (9.5)$$

$$\text{or } Ft = mv - mu$$

That is, when  $F = 0$ ,  $v = u$  for whatever time,  $t$  is taken. This means that the object will continue moving with uniform velocity,  $u$  throughout the time,  $t$ . If  $u$  is zero then  $v$  will also be zero. That is, the object will remain at rest.

**Example 9.1** A constant force acts on an object of mass 5 kg for a duration of 2 s. It increases the object's velocity from  $3 \text{ m s}^{-1}$  to  $7 \text{ m s}^{-1}$ . Find the magnitude of the applied force. Now, if the force was applied for a duration of 5 s, what would be the final velocity of the object?

**Solution:**

We have been given that  $u = 3 \text{ m s}^{-1}$  and  $v = 7 \text{ m s}^{-1}$ ,  $t = 2 \text{ s}$  and  $m = 5 \text{ kg}$ . From Eq. (9.5) we have,

$$F = \frac{m(v-u)}{t}$$

Substitution of values in this relation gives

$$F = 5 \text{ kg } (7 \text{ m s}^{-1} - 3 \text{ m s}^{-1}) / 2 \text{ s} = 10 \text{ N.}$$

Now, if this force is applied for a duration of 5 s ( $t = 5 \text{ s}$ ), then the final velocity can be calculated by rewriting Eq. (9.5) as

$$v = u + \frac{Ft}{m}$$

On substituting the values of  $u$ ,  $F$ ,  $m$  and  $t$ , we get the final velocity,

$$v = 13 \text{ m s}^{-1}.$$

$5 \text{ m s}^{-2}$  would require a greater force.

**Example 9.3** A motorcar is moving with a velocity of  $108 \text{ km/h}$  and it takes  $4 \text{ s}$  to stop after the brakes are applied. Calculate the force exerted by the brakes on the motorcar if its mass along with the passengers is  $1000 \text{ kg}$ .

**Solution:**

The initial velocity of the motorcar  
 $u = 108 \text{ km/h}$   
 $= 108 \times 1000 \text{ m}/(60 \times 60 \text{ s})$   
 $= 30 \text{ m s}^{-1}$

and the final velocity of the motorcar  
 $v = 0 \text{ m s}^{-1}$ .

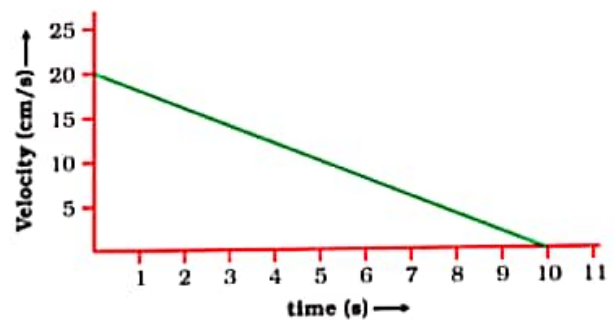
The total mass of the motorcar along with its passengers =  $1000 \text{ kg}$  and the time taken to stop the motorcar,  $t = 4 \text{ s}$ . From Eq. (9.5) we have the magnitude of the force applied by the brakes  $F$  as  $m(v - u)/t$ .

On substituting the values, we get  
 $F = 1000 \text{ kg} \times (0 - 30) \text{ m s}^{-1}/4 \text{ s}$   
 $= -7500 \text{ kg m s}^{-2}$  or  $-7500 \text{ N}$ .

The negative sign tells us that the force exerted by the brakes is opposite to the direction of motion of the motorcar.

**Example 9.4** A force of  $5 \text{ N}$  gives a mass  $m_1$ , an acceleration of  $10 \text{ m s}^{-2}$  and a mass  $m_2$ , an acceleration of  $20 \text{ m s}^{-2}$ . What acceleration would it give if both the masses were tied together?

**Example 9.5** The velocity-time graph of a ball of mass  $20 \text{ g}$  moving along a straight line on a long table is given in Fig. 9.9.



**Fig. 9.9**

How much force does the table exert on the ball to bring it to rest?

**Solution:**

The initial velocity of the ball is  $20 \text{ cm s}^{-1}$ . Due to the friction force exerted by the table, the velocity of the ball decreases down to zero in  $10 \text{ s}$ . Thus,  $u = 20 \text{ cm s}^{-1}$ ;  $v = 0 \text{ cm s}^{-1}$  and  $t = 10 \text{ s}$ . Since the velocity-time graph is a straight line, it is clear that the ball moves with a constant acceleration. The acceleration  $a$  is,

$$\begin{aligned} a &= \frac{v - u}{t} \\ &= (0 \text{ cm s}^{-1} - 20 \text{ cm s}^{-1})/10 \text{ s} \\ &= -2 \text{ cm s}^{-2} = -0.02 \text{ m s}^{-2}. \end{aligned}$$